

**IMPROVED COUPLED CHANNELS AND R -MATRIX MODELS:
 pp PREDICTIONS TO 1 GeV ***

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ABSTRACT

The NN scattering near inelastic threshold is sensitive to the long-range diagonal interaction in the produced isobar channel. Earlier models included meson exchange potentials in the NN sector and connecting that sector to isobar channels, as well as an R -matrix description of short-range quark effects. Including the diagonal pion exchange contributions in the $N\Delta$ channels coupled to the very inelastic 1D_2 and 3F_3 NN channels substantially improves those phase parameters and the observables $\Delta\sigma_L$ and $\Delta\sigma_T$. The same improvement is made to the other $I = 1$ NN channels. In the 1S_0 NN channel coupling to the $\Delta\Delta$ channel is unusually important due to the $L = 2$ angular momentum barrier in the $N\Delta$ channel. The pion exchange transition potentials between isobar channels are included in this partial wave to obtain the correct equilibrium between one- and two-pion decay channels. Other improvements to earlier models have been made, in the specification of isobar channels and the inclusion of decay width effects in more channels. A comparison is made with all the pp data for $T_L \leq 800$ MeV, producing a very good fit over the whole energy range.

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I. Introduction

In describing the nucleon-nucleon (NN) interaction beyond the first 150 MeV of kinetic energy in the center of momentum system (nucleon beam energy $T_L > 300$ MeV), coupling to isobar channels introduces an important energy dependence to the real phase even when the imaginary phase remains small. Single-pion production begins at $T_L = 280$ MeV, but is relatively weak until the Δ production threshold, $T_L = 630$ MeV is approached. But the effect on the elastic scattering begins below the threshold of substantial inelasticity because of the effect of virtual intermediate states of higher mass. Consequently, a coupled channel system must be used to understand the NN reaction in the whole range $0 \leq T_L \leq 800$ MeV. Experience has shown^{1,2,3} that for some NN partial waves the $\Delta\Delta$ and $NN^*(1440)$ channels are important, in addition to the low threshold $N\Delta$ channel.

The R -matrix (or Boundary Condition) model^{4,5} incorporates long-range meson exchange effects in the potential matrix outside the boundary radius r_0 .¹ The internal dynamics, including the effects of quark degrees of freedom, is described by a simple boundary condition at r_0 . The boundary condition is a meromorphic function of energy⁴ whose poles are completely described by the properties of simple quark configurations.^{2,3,6,7} Only the constant term is not given by the internal model.

Using a potential matrix and a boundary condition matrix the coupling to isobaric channels is easily incorporated.¹ With some computational burden, but little formal complication, the width of the isobars is included⁸ by discretizing the Breit–Wigner mass distribution of the isobar, treating each mass as a separate channel. In previous work,^{3,8} this has provided a good quantitative description of the phase parameters of most of the proton-proton (pp) partial waves. The most important exceptions to this quantitative agreement have been the detailed shapes of the inelasticity, η , in the 1D_2 and 3F_3 channels.⁸ The predicted positions, strengths and widths of the structure in these channels are in agreement with experiment, indicating that the structures are due to the effect of the $N\Delta$ threshold and require no significant contribution from quark substructure (the latter is important at somewhat higher energies^{2,3,6,7}). In fact this potential and R -matrix model has been the only one to adequately predict the strong onset of inelasticity in the 3F_3 channel. However,

because of the dominance of these channels in the $\Delta\sigma_L(pp)$ and $\Delta\sigma_T(pp)$ structures, for $600 \text{ MeV} < T_L < 800 \text{ MeV}$,⁹ the detailed shaped of the inelasticity must be fairly precise to reproduce the data in the region.

In the model, the π, η, ρ and ω meson exchanges (and also two-pion exchange) are all included in deriving the potential within the NN sector (Fig. 1(a)–(c)).

Because of the indirect effect of the NN to isobar channel transition (Fig. 1(d)–(e)) on the NN elastic interaction only the π meson exchange transition potentials are included. A two-pion exchange transition potential is only important at distances less than one-third of a pion Compton wavelength in the $NN \rightarrow NN$ reactions, at which distance it is partly masked by the effect of the boundary condition. Nevertheless, a phenomenological two-pion range transition potential is sometimes included, which may also substitute for ρ meson exchange.

The diagonal $N\Delta$ interaction is one stage further removed than the transition interaction from influencing the NN scattering, and was entirely represented by the boundary condition in previous work. However, when the energy dependence of the effect is strong, as it is near the $N\Delta$ threshold, then the short-range boundary condition is not an adequate substitute for the long-range diagonal potential. The long-range interaction shown in Fig. 1(f)–(g) is more important at small momentum transfers. In this paper we include these diagonal potentials, improving many partial wave phases. In particular, the η and δ in the 1D_2 and 3F_3 channels are greatly improved for $T_L > 500 \text{ MeV}$.

In previous work^{2,3} there was some difficulty in obtaining a very good fit to the higher energy 1S_0 NN phases simultaneously with correctly predicting the experimentally very small two-pion production at $T_L = 800 \text{ MeV}$.¹⁰ The $N\Delta$ diagonal potential affects this situation positively, but the transition between $N\Delta$ channels, which produce one pion, and $\Delta\Delta$ or $NN^*(1440)$ channels, which produce two pions, critically affects the pion production multiplicity. The long-range one-pion exchange transition potentials between isobar channels are important to this spectrum at low energies. We include them here in the 1S_0 channel and obtain good results for both elastic NN scattering and pion production.

In previous work some isobar channels differing only in spin were combined. They are now treated as separate channels to more accurately treat the pion exchange potentials and for future

predictions of 3-body final state distributions. The effect of isobar width is included in more channels.

In some instances the one-pion exchange (OPE) coefficients for particular channels were incorrect in previous results. These corrections will be noted in the description of each partial wave. r_0 and $g_{NN\pi}$ have been chosen so that one value is used throughout.

In Section II we review critical aspects of the model. Section III describes the model elements and parameters for each NN partial wave and displays the fit to the phase parameters. In Section IV all the model partial waves with $J \leq 4$, and the one-pion-exchange amplitude for higher J partial waves are used to predict the experimental observables. The excellent results are displayed with full angular distribution of all spin observables for which there is data at key energies spanning $T_L \leq 800$ MeV. In addition, we present excitation curves at 90° for these observables and total cross sections (with and without polarization) over the whole range. In particular, we note that the well-known structures⁹ in $\Delta\sigma_L(pp)$ and $\Delta\sigma_T(pp)$ are well-fitted with the exception of the somewhat shifted position of the $\Delta\sigma_T$ peak. This defect is correlated with a slightly low minimum value of $\eta(^1D_2)$. The physics that may account for this will be discussed.

II. The Hadron Interaction Model

A. The Schrödinger Equation

For the internucleon distance $r > r_0$, the coupled system wave function is determined by a homogeneous boundary condition at r_0 (to be discussed later) and a coupled channel Schrödinger potential with a meson exchange potential matrix. Using the strong interaction symmetries we assume conservation of total angular momentum, J , isospin I and parity. As is well-known this implies total spin, S , conservation in the NN system (neglecting the $n - p$ mass difference). Consequently, the coupled system contains only one ($L = J$) or two ($L = J \pm 1, S = 1$) NN partial waves, and may have as many $N\Delta$, $\Delta\Delta$ or $NN^*(1440)$ partial waves as have the same J, I and parity. Because of the width of the isobars each isobar channel partial wave is treated as many channels,⁸ with the mass distribution discretized and weighted by the Breit–Wigner distribution. For each channel, i , the eigenvalue is determined by the relativistic value of the relative momentum k_i in that channel, where the total energy $W = \sqrt{M_i^{a2} + k_i^2} + \sqrt{M_i^{b2} + k_i^2}$ with M_i^a and M_i^b being

the pair of baryon masses in channel i . The system of Schrödinger equations is described in detail in Ref. [8].

B. The Potential in the NN Sector

Within the NN sector, the potential matrix (diagonal and tensor coupling elements) is determined by the one-boson exchange of π , η , ρ and ω mesons (Fig. 1(a)) and two-pion exchange (Fig. 1(b)–(c)) in the adiabatic limit. These potentials, with the exception of the spin-orbit term, are described in Ref. [5]. In particular, the form of the two-pion potential is detailed in that reference including the “pair suppression” factor and the Breuckner–Watson vs. Taketani–Machida–Ohnuma ambiguity which arises in the adiabatic limit. All the coupling constants (with the exception noted below), the meson masses and the two parameters which arise in two-pion exchange potential are as in Ref. [5]. The exception is a small change in the value of the pseudoscalar coupling $g_{NN\pi}$. As a result of the larger core separation radius now used, and the coupling to isobars, the value now used is $g_{NN\pi}^2/4\pi = 14.40$ (in place of the value 14.94 in Ref. [5]).

The spin-orbit potential, $\vec{L} \cdot \vec{S} V_{LS}$, was ignored in Ref. [5] because of its short range. The tail of this potential beyond r_0 has been found to be of some importance to the $L \neq 0$ partial waves. The sources of this term are two-pion, ρ and ω exchange, so that it has contributions of different range, form (the two-pion part is non-Yukawa) and sign. Because of the relatively small overall effect of this part of the potential, the radial form has been simplified by fitting a single Yukawa potential to the complete, relativistic result of Ref. [11]. In the $I = 1$ states this results in

$$V_{LS} = -7 \frac{e^{-5m_\pi r}}{r} . \quad (2.1)$$

It is smaller in the $I = 0$ states and we neglect it.

C. The Transition Potentials

The off-diagonal components of the potential matrix connecting to isobar channels are obtained from one-pion exchange as shown in Fig. 1(d)–(e). The general form is¹²

$$V_T(r) = \frac{m_\pi}{3} \frac{f_1 f_2}{4\pi} \vec{T}_1 \cdot \vec{T}_2 \left[\vec{S}_1 \cdot \vec{S}_2 v_0(r) + S_{12}^T v_2(r) \right] \quad (2.2)$$

where the f_i are the coupling constants at vertex i , the \vec{S}_i (\vec{T}_i) are spin (isospin) or transition spin (isospin) operators at vertex i that will be defined below, the tensor operator is

$$S_{12}^T = 3 \left(\vec{S}_1 \cdot \hat{r} \right) \left(\vec{S}_2 \cdot \hat{r} \right) - \vec{S}_1 \cdot \vec{S}_2 , \quad (2.3)$$

$$v_0(r) = \frac{e^{-m_\pi r}}{m_\pi r} \quad \text{and} \quad v_2(r) = \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2} \right) v_0(r) . \quad (2.4)$$

The matrix elements of the above operators are expressible as^{12, 13}

$$\begin{aligned} & \langle S'_1 S'_2 S' L' J | \vec{S}_1 \cdot \vec{S}_2 | S_1 S_2 S L J \rangle \\ &= (-1)^{S_1 + S'_2 + S} \delta_{SS'} \delta_{LL'} \left\{ \begin{array}{ccc} S & S'_2 & S'_1 \\ 1 & S_1 & S_2 \end{array} \right\} (S'_1 \| \vec{S}_1 \| S_1) (S'_2 \| \vec{S}_2 \| S_2) , \end{aligned} \quad (2.5)$$

the analogous expression for $\vec{T}_1 \cdot \vec{T}_2$ and

$$\begin{aligned} & \langle S'_1 S'_2 S' L' J | S_{12}^T | S_1 S_2 S L J \rangle = (-1)^{S'+J} [30(2L+1)(2L'+1)(2S+1)(2S'+1)]^{1/2} \\ & \times \left\{ \begin{array}{ccc} J & S' & L' \\ 2 & L & S \end{array} \right\} \left(\begin{array}{ccc} L' & 2 & L \\ 0 & 0 & 0 \end{array} \right) \left\{ \begin{array}{ccc} S'_1 & S_1 & 1 \\ S'_2 & S_2 & 1 \\ S' & S & 2 \end{array} \right\} (S'_1 \| \vec{S}_1 \| S_1) (S'_2 \| \vec{S}_2 \| S_2) \end{aligned} \quad (2.6)$$

where the standard notation is used for the $3j$ (Wigner), $6j$ (Racah) and $9j$ coefficients. The Wigner–Eckart theorem requires¹³

$$\langle S'm' | S^M | S, m \rangle = N \langle S'm' | 1M, Sm \rangle \quad (2.7)$$

where the normalization N is determined by convention and the $\langle S'm' | JM, Sm \rangle$ are Clebsch–Gordan coefficients. In the Edmonds convention¹⁴ $N = 1$ for the transition spin operators. For those cases diagonal in spin magnitude, because the Pauli spin operators and the equivalent spin- $3/2$ operator are twice the standard angular momentum operator normalization $\vec{J}^2 = J(J+1)$, $N = -2\sqrt{S(S+1)}$. For $NN\pi$, $NN^*\pi$, $N\Delta\pi$ and $\Delta\Delta\pi$ vertices this requires

$$\left(\frac{1}{2} \| \vec{\sigma} \| \frac{1}{2} \right) = \sqrt{6} , \quad (2.8a)$$

$$\left(\frac{3}{2} \| \vec{S}_{1/2,3/2} \| \frac{1}{2} \right) = - \left(\frac{1}{2} \| \vec{S}_{3/2,1/2} \| \frac{3}{2} \right) = 2 \quad (2.8b)$$

and

$$\left(\frac{3}{2} \| \vec{\Sigma} \| \frac{3}{2} \right) = 2\sqrt{15} \quad (2.8c)$$

respectively, with analogous results for the isospin operators.

With these normalizations of the operators we have

$$f_{NN\pi} = \frac{m_\pi}{2M} g_{NN\pi} \quad (2.9)$$

where in pp scattering, m_π is the neutral pion mass and M is the proton mass. As discussed in Ref. [1], we obtain $f_{N\Delta\pi}$ from the decay width of the Δ , giving $f_{N\Delta\pi}^2/4\pi = 0.35$ which is 50% larger than the quark model value but only 5% smaller than the result of a strong coupling model.¹² Similarly, we obtain¹ $f_{NN^*\pi}$ from the $N^*(1440)$ decay to a single pion, giving $f_{NN^*\pi}^2/4\pi = 0.015$. Lacking other information we have used the consistent quark and strong coupling model result¹²

$$\begin{aligned} f_{\Delta\Delta\pi} &= \frac{1}{5} f_{NN\pi} , \\ f_{\Delta\Delta\pi}^2/4\pi &= 0.003 . \end{aligned} \quad (2.10)$$

D. The Diagonal $N\Delta$ Potential

The equations and matrix elements for the diagonal $N\Delta$ potential (see Fig. 1(f)–(g)) are of the same form as above in Section IIC. Here we discuss the implications of the fact that there are contributions from two distinct Feynman diagrams: the direct interactions, Fig. 1(f), and the exchange interaction, Fig. 1(g). To obtain its contribution to $V_{N\Delta,N\Delta}$, the exchange amplitude must be multiplied by $(-1)^{L+S+T+1}$.

First we note that the coupling constant product for the direct interaction is $f_{NN\pi} f_{\Delta\Delta\pi}/4\pi = 0.0155$, while that for the exchange interaction is $f_{N\Delta\pi}^2/4\pi = 0.351$. However, that apparent dominance of the exchange interaction is largely reversed by the difference of the spin matrix elements. In the isotriplet state the matrix element of $\vec{\tau} \cdot \vec{T}$ is -5 , but the matrix element of $\vec{T}_{1/2,3/2} \cdot \vec{T}_{3/2,1/2}$ is $-\frac{1}{3}$. In addition, the spin-spin and tensor matrix elements are in most states larger for the direct interaction. The result is that the two contributions are of the same order, and as they sometimes differ in sign, the strength and sign of the long-range diagonal $N\Delta$ interaction is very sensitive to the channel involved. One may then hope to determine the effect of the diagonal coupling by comparing the effect in different channels. As we shall see, this is in fact the case in the 1D_2 and 3F_3 channels, where strong inelasticity enhances the sensitivity to the diagonal components.

E. The R -Matrix Boundary Condition

As shown in Ref. [4], the boundary condition at r_0 is of the general form (α, β are channel indices)

$$R_0 \frac{d\psi_\alpha^W}{dr_0} = \sum_\beta f_{\alpha\beta}(W) \psi_\beta^W(r) \quad (2.11)$$

where W is the total barycentric energy and the $f_{\alpha\beta}(W)$ are meromorphic functions of W with poles of positive residue on the real axis. As discussed in Refs. [4] and [15], these poles are determined by the complete set of states in the interior satisfying $\psi_{\text{int}}(r_0) = 0$. The pole position is given by the eigenvalue W_i of the corresponding interior state, and the residue by the derivatives of the interior wavefunctions at r_0 . It was shown in Ref. [16] that the residues can be re-expressed in terms of the rate of change of eigenvalue with r_0

$$\rho_{\alpha\beta}^i = -r_0 \frac{\partial W_i}{\partial r_0} \xi_\alpha^i \xi_\beta^i \quad (2.12)$$

where the ξ_α^i are the fractional parentage coefficients of the channel α in the interior state i .

We then have

$$f_{\alpha\beta}(W) = f_{\alpha\beta}^0 + \sum_i \frac{\rho_{\alpha\beta}^i}{W - W_i} . \quad (2.13)$$

The fractional parentage coefficients relevant to our models are those of the $(^1S_{1/2})^6$ quark configuration which overlaps with the $NN(^1S_0)$, $\Delta\Delta(^1S_0)$, $N\Delta(^5S_2)$ and $\Delta\Delta(^5S_2)$ channels. They are $\xi_{NN}^{0+} = \frac{1}{3}$, $\xi_{\Delta\Delta}^{0+} = \frac{2}{\sqrt{45}}$, $\xi_{N\Delta}^{2+} = \frac{1}{\sqrt{6}}$ and $\xi_{\Delta\Delta}^{2+} = \frac{1}{\sqrt{30}}$.

References [2] and [3] discuss the theoretical constraints on r_0 , and the experimental determination $r_0 = 1.05$ fm when the quark interior is described by the Cloudy Bag Model (CBM) parameters. In the CBM, in contrast to the MIT Bag Model, the experimentally determined r_0 is consistent with the theoretical constraints.

III. Isobar Channels, Meson Exchange Potentials, Quark Configurations and Parameters of the pp Partial Waves

For $J > 4$ the amplitude used is that of OPE and the Coulomb force. Below we give the details of the model for each NN , $I = 1$ partial wave with $J \leq 4$.

There are several properties of the model that are common to all the partial wave models. The R -matrix separation radius, r_0 , is determined by requiring that the $[q(1S_{1/2})]^6$ configuration

energy, as determined by the CBM, be the energy for which the inner zero of the external wave function is at r_0 . The external wave function is determined by the potential matrix and the S -matrix. The data determines the latter for $T_L < 1$ GeV. For the 1S_0 and 3S_1 – 3D_1 states the CBM and the data require² $r_0 = 1.05$ fm. The same value is indicated for the 1D_2 state.³ This value is about 20% smaller than the value of r corresponding to the equilibrium radius of these six-quark bags, which satisfies the R -matrix method requirement that r_0 be within the region of asymptotic freedom.² We use $r_0 = 1.05$ fm for all partial waves.

The coupling constants used at the meson-baryon vertices are those given in Refs. [1] and [5], with the exception that we use $g_{NN\pi}^2/4\pi = 14.40$ in all partial waves. This change is related to the larger r_0 now being used. One other consistent change from the earlier published work is an extra factor $\sqrt{2}$ in the coefficients of the $NN \rightarrow N\Delta$ potentials. This is required by contribution of the $NN \rightarrow \Delta N$ diagram, which is of the same strength as that of the $NN \rightarrow N\Delta$ diagram.

Table I presents the values of all the isospin-isospin, spin-spin and tensor matrix elements that are needed for the determination of the OPE potentials between NN and isobar channels or between isobar channels, for isobar channels with one or two Δ 's. The specific numerical coefficients are given in the subsection for each partial wave.

We note that the f -matrix parameters determined by fitting the data depend slightly on the number and distribution of the discretized mass spectrum of the isobars. We have used 17 channels for each isobar whose width is not neglected. Other aspects of the distribution are discussed in Ref. [8].

A. The NN (1S_0) System

The last published versions of the coupled-channel R -matrix model for this system were the “new S ” model of Ref. [3] and the “old S ” model.^{2,3} Those versions were the first to include the CBM pole in the f -matrix with the associated larger value $r_0 = 1.05$ fm. The “new S ” version takes isobar width into account. It was consistent with the experimental value of $\sigma(pp \rightarrow pp\pi^+\pi^-)$ at $T_L = 800$ MeV. Its only important discrepancy from experiment was that it predicted too much inelasticity for $600 \text{ MeV} < T_L \leq 800 \text{ MeV}$. It predicted $\eta(800 \text{ MeV}) = 0.84$ while the phase shift analysis (PSA) prefers 0.98. The excess of inelasticity was a result of the strength of coupling to the

Table I.

Spin-spin and tensor matrix element for NN - $N\Delta$, NN - $\Delta\Delta$, $N\Delta$ - $N\Delta$ and $N\Delta$ - $\Delta\Delta$ transitions.
 α and α' represent the sets of quantum numbers in the first two columns.

J	L	S_1	S_2	S	J	L'	S'_1	S'_2	S'	$\langle \alpha' \vec{S}_1 \cdot \vec{S}_2 \alpha \rangle$	$\langle \alpha' S_{12}^T \alpha \rangle$
0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	2	$\frac{1}{2}$	$\frac{3}{2}$	2	0	$+\sqrt{6}$
	"				0	0	$\frac{3}{2}$	$\frac{3}{2}$	0	$-\sqrt{2}$	0
	"				0	2	$\frac{3}{2}$	$\frac{3}{2}$	2	0	$\sqrt{2}$
0	2	$\frac{1}{2}$	$\frac{3}{2}$	2	0	0	$\frac{3}{2}$	$\frac{3}{2}$	0	0	$+\sqrt{3}$
	"				0	2	$\frac{3}{2}$	$\frac{3}{2}$	2	$-2\sqrt{3}$	$\sqrt{3}$
1	0	$\frac{1}{2}$	$\frac{3}{2}$	1	1	0	$\frac{3}{2}$	$\frac{3}{2}$	1	$-2\sqrt{5/3}$	—
0	2	$\frac{1}{2}$	$\frac{3}{2}$	2	0	2	$\frac{1}{2}$	$\frac{3}{2}$	2	3	-6
	"				0	2	$\frac{3}{2}$	$\frac{1}{2}$	2	+1	1
0	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0	1	$\frac{1}{2}$	$\frac{3}{2}$	1	$+\sqrt{8/3}$	$+\sqrt{2/3}$
	"				0	1	$\frac{3}{2}$	$\frac{3}{2}$	1	$-\sqrt{10/3}$	$\frac{2}{3}\sqrt{2/5}$
0	1	$\frac{1}{2}$	$\frac{3}{2}$	1	0	1	$\frac{1}{2}$	$\frac{3}{2}$	1	-5	2
	"				0	1	$\frac{3}{2}$	$\frac{1}{2}$	1	$+\frac{1}{3}$	$\frac{5}{3}$
1	1	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	$\frac{1}{2}$	$\frac{3}{2}$	1	$+\sqrt{8/3}$	$-\sqrt{1/6}$
	"				1	1	$\frac{1}{2}$	$\frac{3}{2}$	2	0	$3\sqrt{3/10}$
	"				1	3	$\frac{1}{2}$	$\frac{3}{2}$	2	0	$3/\sqrt{5}$
	"				1	1	$\frac{3}{2}$	$\frac{3}{2}$	1	$-\sqrt{10/3}$	$-\frac{1}{3}\sqrt{2/5}$
1	1	$\frac{1}{2}$	$\frac{3}{2}$	1	1	1	$\frac{1}{2}$	$\frac{3}{2}$	1	-5	-1
	"				1	1	$\frac{3}{2}$	$\frac{1}{2}$	1	$+\frac{1}{3}$	$-\frac{5}{6}$
1	1	$\frac{1}{2}$	$\frac{3}{2}$	2	1	1	$\frac{1}{2}$	$\frac{3}{2}$	2	3	$-\frac{21}{5}$
	"				1	1	$\frac{3}{2}$	$\frac{1}{2}$	2	+1	$\frac{7}{10}$
1	3	$\frac{1}{2}$	$\frac{3}{2}$	2	1	3	$\frac{1}{2}$	$\frac{3}{2}$	2	3	$-\frac{24}{5}$
	"				1	3	$\frac{3}{2}$	$\frac{1}{2}$	2	+1	$\frac{4}{5}$
2	1	$\frac{1}{2}$	$\frac{1}{2}$	1	2	1	$\frac{1}{2}$	$\frac{3}{2}$	1	$+\sqrt{8/3}$	$+1/(5\sqrt{6})$
	"				2	1	$\frac{1}{2}$	$\frac{3}{2}$	1	0	$-\frac{3}{5}\sqrt{3/2}$
	"				2	3	$\frac{1}{2}$	$\frac{3}{2}$	1	0	$-\frac{3}{5}$
	"				2	3	$\frac{1}{2}$	$\frac{3}{2}$	2	0	$\frac{3}{5}\sqrt{6}$
2	3	$\frac{1}{2}$	$\frac{1}{2}$	1	2	1	$\frac{1}{2}$	$\frac{3}{2}$	1	0	$-\frac{3}{5}$
	"				2	1	$\frac{1}{2}$	$\frac{3}{2}$	2	0	$-\frac{3}{5}$
	"				2	3	$\frac{1}{2}$	$\frac{3}{2}$	1	$+\sqrt{8/3}$	$\frac{2}{5}\sqrt{2/3}$
	"				2	3	$\frac{1}{2}$	$\frac{3}{2}$	2	0	$\frac{6}{5}$
2	1	$\frac{1}{2}$	$\frac{3}{2}$	1	2	1	$\frac{1}{2}$	$\frac{3}{2}$	1	-5	$\frac{1}{5}$
	"				2	1	$\frac{3}{2}$	$\frac{1}{2}$	1	$+\frac{1}{3}$	1/6

Table I. contd.

J	L	S_1	S_2	S	J'	L'	S'_1	S'_2	S'	$\langle \alpha' \vec{S}_1 \cdot \vec{S}_2 \alpha \rangle$	$\langle \alpha' S_{12}^T \alpha \rangle$
2	1	$\frac{1}{2}$	$\frac{3}{2}$	2	2	1	$\frac{1}{2}$	$\frac{3}{2}$	2	3	$\frac{21}{5}$
"					2	1	$\frac{3}{2}$	$\frac{1}{2}$	2	+1	$-\frac{7}{10}$
2	3	$\frac{1}{2}$	$\frac{3}{2}$	1	2	3	$\frac{1}{2}$	$\frac{3}{2}$	1	-5	$\frac{4}{5}$
"					2	3	$\frac{3}{2}$	$\frac{1}{2}$	1	$+\frac{1}{3}$	$\frac{2}{3}$
2	3	$\frac{1}{2}$	$\frac{3}{2}$	2	2	3	$\frac{1}{2}$	$\frac{3}{2}$	2	3	$-\frac{6}{5}$
"					2	2	$\frac{3}{2}$	$\frac{1}{2}$	2	+1	$\frac{1}{5}$
2	2	$\frac{1}{2}$	$\frac{1}{2}$	0	2	0	$\frac{1}{2}$	$\frac{3}{2}$	2	0	$\sqrt{6/5}$
"					2	2	$\frac{1}{2}$	$\frac{3}{2}$	2	0	$-2\sqrt{3}$
"					2	0	$\frac{3}{2}$	$\frac{3}{2}$	2	0	$\sqrt{2/5}$
2	0	$\frac{1}{2}$	$\frac{3}{2}$	2	2	0	$\frac{1}{2}$	$\frac{3}{2}$	2	3	0
"					2	0	$\frac{3}{2}$	$\frac{1}{2}$	2	+1	0
2	2	$\frac{1}{2}$	$\frac{3}{2}$	2	2	2	$\frac{1}{2}$	$\frac{3}{2}$	2	3	$\frac{9}{7}$
3	3	$\frac{1}{2}$	$\frac{1}{2}$	1	3	1	$\frac{1}{2}$	$\frac{3}{2}$	2	+1	$-\frac{3}{14}$
"					3	3	$\frac{1}{2}$	$\frac{3}{2}$	1	$+\sqrt{8/3}$	$-1/\sqrt{6}$
"					3	3	$\frac{1}{2}$	$\frac{3}{2}$	2	0	$\sqrt{3/10}$
3	1	$\frac{1}{2}$	$\frac{3}{2}$	2	3	1	$\frac{1}{2}$	$\frac{3}{2}$	2	3	$-\frac{6}{5}$
"					3	1	$\frac{3}{2}$	$\frac{1}{2}$	2	+1	$\frac{1}{5}$
3	3	$\frac{1}{2}$	$\frac{3}{2}$	1	3	3	$\frac{1}{2}$	$\frac{3}{2}$	1	-5	-1
"					3	3	$\frac{3}{2}$	$\frac{1}{2}$	2	+1	$\frac{1}{5}$
3	3	$\frac{1}{2}$	$\frac{3}{2}$	2	3	3	$\frac{1}{2}$	$\frac{3}{2}$	2	3	$14\sqrt{2}/9$
"					3	3	$\frac{3}{2}$	$\frac{1}{2}$	2	1	$-7\sqrt{2}/27$
4	3	$\frac{1}{2}$	$\frac{1}{2}$	1	4	3	$\frac{1}{2}$	$\frac{3}{2}$	2	0	$-\sqrt{5/6}$
"					4	3	$\frac{1}{2}$	$\frac{3}{2}$	1	$+\sqrt{8/3}$	$1/(3\sqrt{6})$
4	5	$\frac{1}{2}$	$\frac{1}{2}$	1	4	3	$\frac{1}{2}$	$\frac{3}{2}$	2	0	$-\sqrt{2/3}$
"					4	3	$\frac{1}{2}$	$\frac{3}{2}$	1	0	$-\frac{1}{3}\sqrt{10/3}$
4	3	$\frac{1}{2}$	$\frac{3}{2}$	2	4	3	$\frac{1}{2}$	$\frac{3}{2}$	2	3	3
"					4	3	$\frac{3}{2}$	$\frac{1}{2}$	1	+1	$-\frac{1}{2}$
4	3	$\frac{1}{2}$	$\frac{3}{2}$	1	4	3	$\frac{1}{2}$	$\frac{3}{2}$	1	-5	$\frac{1}{3}$
"					4	3	$\frac{3}{2}$	$\frac{1}{2}$	1	$+1/3$	$\frac{5}{18}$
4	4	$\frac{1}{2}$	$\frac{1}{2}$	0	4	2	$\frac{1}{2}$	$\frac{3}{2}$	2	3	$-\frac{12}{7}$
4	2	$\frac{1}{2}$	$\frac{3}{2}$	2	4	2	$\frac{1}{2}$	$\frac{3}{2}$	2	3	$-\frac{12}{7}$
"					4	2	$\frac{3}{2}$	$\frac{1}{2}$	2	+1	$\frac{2}{7}$

$N\Delta$ channel, which in turn was required to produce the correct energy dependence of the real phase δ . Switching coupling strength to the high threshold $\Delta\Delta$ or NN^* channels reduced the inelasticity, but produced excessive two-pion production as in the “old S ” model.

The OPE potential in the diagonal $N\Delta$ interaction, now included, is repulsive, decreasing the inelasticity near the $N\Delta$ threshold. Because of the importance of two-pion production we also include the OPE transition potentials between the $N\Delta$ and $\Delta\Delta$ states. We can now transfer $NN - N\Delta$ coupling strength to $NN - \Delta\Delta$ coupling, but the increased production of these two-pion channels is transferred back to one-pion channels by the $N\Delta - \Delta\Delta$ coupling.

The $\Delta\Delta ({}^5D_0)$ channel is now included along with the $\Delta\Delta ({}^1S_0)$ channel, because of the former’s strong OPE coupling. But the width of this channel is neglected, because its orbital angular momentum barrier means that its effective threshold is higher than its mass threshold. On the other hand, the $NN^* ({}^1S_0)$ channel has been dropped because of its weak OPE coupling to the NN channel. It would be difficult to distinguish its effect from that of the $\Delta\Delta ({}^1S_0)$ channel. The result is a good fit to both δ and η over the whole energy range. Simultaneously the production of the two-pion producing $\Delta\Delta$ channel is kept low enough so that the contribution to $\sigma (pp \rightarrow pp\pi^+\pi^-)$ is about one-third of the experimental total value¹⁰ of 3 μb for $T_L = 800$ MeV.

The potential matrix elements involving isobar channels are (Eq. (2.2))

$$\begin{aligned} V_T [NN ({}^1S_0), N\Delta ({}^5D_0)] &= 0.311m_\pi v_2 \\ V_T [NN ({}^1S_0), \Delta\Delta ({}^1S_0)] &= 0.172m_\pi v_0 \\ V_T [NN ({}^1S_0), \Delta\Delta ({}^5D_0)] &= -0.1744m_\pi v_2 \\ V_T [N\Delta ({}^5D_0), \Delta\Delta ({}^1S_0)] &= -0.0696m_\pi v_2 \\ V_T [N\Delta ({}^5D_0), \Delta\Delta ({}^5D_0)] &= 0.139m_\pi v_0 - 0.0696m_\pi v_2 \quad \text{and} \\ V_D [N\Delta ({}^5D_0)] &= -0.0383v_0 + 0.1935v_2 . \end{aligned}$$

We note that the direct and exchange contributions to V_D (Figs. 3a and 3b) cancel in the coefficient of v_0 , but add in the coefficient of v_2 producing a substantial repulsive potential.

The pole of the 0^+ , $[q ({}^1S_{1/2})]^6$ configuration is at $W_p = 2.71$ GeV/ c^2 , corresponding to $T_L = 2.04$ GeV. Only the $\Delta\Delta ({}^1S_0)$ and $NN ({}^1S_0)$ channels overlap with the configuration, and

they have residues (Eq. (2.12))

$$\begin{aligned}\rho_{NN,NN} &= 0.149 \text{ GeV} , \\ \rho_{NN,\Delta\Delta 0} &= 0.133 \text{ GeV} \quad \text{and} \\ \rho_{\Delta\Delta 0,\Delta\Delta 0} &= 0.118718 \text{ GeV} .\end{aligned}$$

Here and below we use the total spin S of the channel in the subscripts to distinguish otherwise ambiguous channels.

The constant, non-vanishing, elements of the f -matrix are adjusted to fit the phase parameters and the singlet scattering length $a_s^{pp} = -7.82$ fm. They are $f_{NN,NN}^0 = 83.5743$, $f_{N\Delta,N\Delta}^0 = 0.0$, $f_{\Delta\Delta 0,\Delta\Delta 0}^0 = 3.0$, $f_{\Delta\Delta 2,\Delta\Delta 2}^0 = 5.0$, $f_{N\Delta,N\Delta}^0 = 6.0$ and $f_{NN,\Delta\Delta 2}^0 = -25.2$. The resulting phases are compared with the PSA results^{17,18} in Fig. 2, showing a very good fit. The PSA results of Refs. [19,20] generally agree with those of Refs. [17,18] but Ref. [19] does not go to the lower energies, and Ref. [20] has oscillations not present in the other PSA results or in ours.

B. The $NN(^3P_0)$ System

In the last published version⁸ the $NN(^3P_0)$ channel was coupled to the $N\Delta(^3P_0)$ channel only. We now also include the $\Delta\Delta(^3P_0)$ but neglect the effect of its width. In Ref. [8] the phase shift fit was improved for $T_L < 300$ MeV by reducing the pion coupling to $g_{NN\pi}^2/4\pi = 13.0$. However, with the increased value of r_0 used now (1.05 fm instead of 0.75 fm) we fit well with $g_{NN\pi}^2/4\pi = 14.4$ as in all the other partial waves. The added diagonal OPE potential for $N\Delta$ is repulsive.

The potential matrix elements are

$$\begin{aligned}V_T [NN(^3P_0), N\Delta(^3P_0)] &= 0.2075m_\pi v_0 + 0.1037m_\pi v_2 \\ V_T [NN(^3P_0), \Delta\Delta(^3P_0)] &= 0.130m_\pi v_0 - 0.052m_\pi v_2 \quad \text{and} \\ V_D [N\Delta(^3P_0)] &= 0.1418m_\pi v_0 + 0.0135m_\pi v_2 .\end{aligned}$$

As seen in Fig. 3, a very good fit to the PSA is obtained with the non-zero f -matrix elements $f_{NN,NN}^0 = 11.7$, $f_{N\Delta,N\Delta}^0 = 2.0$, $f_{\Delta\Delta,\Delta\Delta}^0 = 2.0$, $f_{N\Delta,N\Delta}^0 = 1.2$, and $f_{NN,\Delta\Delta}^0 = -6.5$.

We note that, both in our model results and the most recent PSA, the value of η monotonically decreases below 1 GeV, unlike those of Ref. [8]. This is in part due to the repulsive diagonal OPE potential in $N\Delta$. The result is that the previous indication of a resonance in the Argand plot (a counter-clockwise motion) is now gone for $T_L < 1$ GeV.

C. The $NN(^3P_1)$ System

Reference [1] contains the last published version of the $NN(^3P_1)$ system. It uses the smaller r_0 and was coupled to a single $N\Delta(P)$ channel. The width of the $N\Delta$ channel could then only be taken into account when V_T was ignored. We now couple to the $N\Delta(^3P_1)$, $N\Delta(^5P_1)$, $N\Delta(^5F_1)$ and the $\Delta\Delta(^3P_1)$ channels. The width is taken into account only for the $N\Delta(^5P_1)$ channel, which is strongly core coupled as well as having a low angular momentum barrier and a low mass threshold. The other additions and changes common to all partial waves have also been made. The resulting potential matrix elements are

$$\begin{aligned} V_T [NN(^3P_1), N\Delta(^3P_1)] &= 0.2075m_\pi v_0 - 0.05186m_\pi v_2 + 0.8 \frac{e^{-2m_\pi r}}{r} \\ V_T [NN(^3P_1), N\Delta(^5P_1)] &= -0.2087m_\pi v_2 + 3.0 \frac{e^{-2m_\pi r}}{r} \\ V_T [NN(^3P_1), N\Delta(^5F_1)] &= 0.1704m_\pi v_2 - 2.5 \frac{e^{-2m_\pi r}}{r} \\ V_T [NN(^3P_1), \Delta\Delta(^3P_1)] &= 0.13m_\pi v_0 + 0.026m_\pi v_2 \\ V_D [N\Delta(^3P_1)] &= 0.1418m_\pi v_0 - 0.00675m_\pi v_2 \\ V_D [N\Delta(^5P_1)] &= -0.1163m_\pi v_0 + 0.0809m_\pi v_2 \\ V_D [N\Delta(^5F_1)] &= -0.1163m_\pi v_0 + 0.924m_\pi v_2 . \end{aligned}$$

In this case, fitting the PSA required the addition of the phenomenological two-pion range transition potentials to the OPE components connecting NN to $N\Delta$ channels. These phenomenological components substitute for ρ -exchange tails as well as actual two-pion exchange. They usually, as here, decrease the coupling strength of OPE. Although their numerical coefficients are large, because of the difference in the r -dependent form, they are dominated by the OPE tensor terms.

The fitted non-zero f -matrix elements are (as before the numerals 1 and 2 in the subscripts denote the spin for P states, while the 3 denotes the F state) $f_{NN,NN}^0 = 17.915$, $f_{N\Delta 1,N\Delta 1}^0 = 2.0$, $f_{N\Delta 2,N\Delta 2}^0 = 10.0$, $f_{N\Delta 3,N\Delta 3}^0 = 4.0$, $f_{\Delta\Delta,\Delta\Delta}^0 = 5.0$, $f_{NN,N\Delta 2}^0 = -4.0$, and $f_{NN,\Delta\Delta}^0 = 10.85$. As shown in Fig. 4, the fits to PSA are very good.

D. The $NN\left(^3P_2 - ^3F_2\right)$ System

The last published version of the $NN\left(^3P_2 - ^3F_2\right)$ system⁸ already includes the effect of the width of the isobars. In addition to the general changes made to OPE couplings and the diagonal $N\Delta$ potential, the $N\Delta\left(^3P_2, ^5P_2, ^3F_2\right)$ and 5F_2 channels are each now treated independently, without combining the two P states and the two F states. The effect of width in the two F states is neglected because of the angular momentum barrier and the vanishing f -matrix coupling to the NN states. The fit to the PSA is better than in Ref. [8] and no phenomenological two-pion range potentials are now needed. The potentials involving isobars are

$$\begin{aligned} V_T \left[NN\left(^3P_2\right), N\Delta\left(^3P_2\right) \right] &= 0.2075v_0 + 0.01037v_2 \\ V_T \left[NN\left(^3F_2\right), N\Delta\left(^3P_2\right) \right] &= -0.0762v_2 \\ V_T \left[NN\left(^3P_2\right), N\Delta\left(^5P_2\right) \right] &= -0.0933v_2 \\ V_T \left[NN\left(^3F_2\right), N\Delta\left(^5P_2\right) \right] &= 0.0762v_2 \\ V_T \left[NN\left(^3P_2\right), N\Delta\left(^3F_2\right) \right] &= -0.0762v_2 \\ V_T \left[NN\left(^3F_2\right), N\Delta\left(^3F_2\right) \right] &= 0.2075v_0 + 0.0415v_2 \\ V_T \left[NN\left(^3P_2\right), N\Delta\left(^5F_2\right) \right] &= 0.1867v_2 \\ V_T \left[NN\left(^3F_2\right), N\Delta\left(^5F_2\right) \right] &= 0.1525v_2 \\ V_D \left[N\Delta\left(^3P_2\right) \right] &= 0.1418v_0 + 0.0014v_2 \\ V_D \left[N\Delta\left(^5P_2\right) \right] &= -0.1163u_0 - 0.0809v_2 \\ V_D \left[N\Delta\left(^3F_2\right) \right] &= 0.1418v_0 + 0.0054v_2 \\ V_D \left[N\Delta\left(^5F_2\right) \right] &= -0.1163v_0 + 0.02314v_2 \end{aligned}$$

The f -matrix choice to fit the PSA is, for the non-vanishing elements, $f_{NNP,NNP}^0 = 13.917$, $f_{NNF,NNF}^0 = 12.0$, $f_{N\Delta P1,N\Delta P1}^0 = 5.5$, $f_{N\Delta P2,N\Delta P2}^0 = 2.5$, $f_{N\Delta F1,N\Delta F1}^0 = 4.0$, $f_{N\Delta F2,N\Delta F2}^0 = 4.0$, $f_{NNP,NNF}^0 = -2.8$, $f_{NNP,N\Delta P1}^0 = 10.30$, and $f_{NNP,N\Delta P2}^0 = -1.2$.

The fit to the PSA shown in Fig. 5 is a substantial improvement to that of Ref. [8]. For two phase parameters it is not as good a fit as we achieve in most of the partial waves, but is adequate in most respects. There is little significance to the large difference between our ϕ_2 and that of Arndt *et al.*¹⁸ That parameter is very sensitive to the data and is strongly constrained in Ref. [18] by their

assumption of only one coupled isobar channel.

E. The $NN(^1D_2)$ System

In Ref. [3] the f -matrix pole arising from the 2^+ state of the $\left[q\left(^1S_{1/2}\right)\right]^6$ graph configuration was included. The configuration energy, as given by the CBM quark dynamics and $r_0 = 1.05$ fm, is $W_p = 2.880$ GeV. The configuration contains components of the $N\Delta(^5S_2)$ and $\Delta\Delta(^5S_2)$ systems, and the non-vanishing residue-matrix components are (Eq. (2.12))

$$\begin{aligned}\rho_{N\Delta 2, N\Delta 2} &= 0.260 \text{ GeV} \\ \rho_{N\Delta 2, \Delta\Delta 2} &= 0.116276 \text{ GeV} \quad \text{and} \\ \rho_{\Delta\Delta 2, \Delta\Delta 2} &= 0.052 \text{ GeV} .\end{aligned}$$

The rest of this previous model was described in Ref. [8], where the width of the Δ is taken into account. The OPE coupling to the $\Delta\Delta(^5S_2)$ channel was neglected then.

Because its vanishing angular momentum barrier compensates for the higher threshold, we now include the $\Delta\Delta(^5S_2)$ channel as well as the $N\Delta(^5D_2)$ and $N\Delta(^5S_2)$ channels included in Ref. [8]. The width of the $\Delta\Delta$ channel is neglected. Among the general coupling changes made, the most important here is the inclusion of OPE diagonal potentials in the $N\Delta$ channels. They are attractive, greatly improving the shape of the inelasticity parameter, as will be shown below. The required potentials are

$$\begin{aligned}V_T \left[NN(^1D_2), N\Delta(^5S_2) \right] &= 0.1392v_2 \\ V_T \left[NN(^1D_2), N\Delta(^5D_2) \right] &= -0.1663v_2 \\ V_T \left[NN(^1D_2), \Delta\Delta(^5S_2) \right] &= -0.0794v_2 \\ V_D \left[N\Delta(^5S_2) \right] &= -0.0383v_0 \quad \text{and} \\ V_D \left[N\Delta(^5D_2) \right] &= -0.0383v_0 - 0.0415v_2 .\end{aligned}$$

There are no phenomenological two-pion range potentials. The adjusted f -matrix elements are $f_{NN,NN}^0 = 15.60$, $f_{N\Delta S,N\Delta S}^0 = 1.0$, $f_{N\Delta D,N\Delta D}^0 = 10.0$, $f_{\Delta\Delta,\Delta\Delta}^0 = 1.0$, $f_{N\Delta,N\Delta S}^0 = 0.6$, and $f_{NN,\Delta\Delta}^0 = -6.50$.

The resulting fit, shown in Fig. 6, is quite good for δ . For η , in contrast to the fit of Ref. [8], the rapid drop to $T_L = 650$ MeV followed by a sudden leveling is reproduced. The remaining defect is that the model value of η in the 700 MeV region is too small by about 0.06 and the model value

increases beyond 800 MeV. It is the attractive OPE diagonal $N\Delta$ potential which increases the effect of coupling at the lower energies, but is less effective at the higher energies, as required.

F. The $NN(^3F_3)$ System

The Ref. [8] version of the $NN(^3F_3)$ system, the last previously published, has the opposite deficiency to that just encountered in the 1D_2 state with respect to the inelasticity. The experimental η keeps decreasing uniformly for $515 \text{ MeV} < T_L < 800 \text{ MeV}$, while the model η begins to level off near 650 MeV. The OPE diagonal interaction in the $N\Delta(^5P_3)$ channel is dominantly repulsive (see below). The rapid drop of η above threshold can be maintained by increasing the coupling to the NN channel, and the drop is then maintained to higher energy as the long-range repulsive potential becomes less effective.

We now also include the $N\Delta(^3F_3)$ and $N\Delta(^5F_3)$ channels. The width of all $N\Delta$ channels is considered. The potentials are

$$\begin{aligned} V_T [NN(^3F_3), N\Delta(^5P_3)] &= -0.1822v_2 \\ V_T [NN(^3F_3), N\Delta(^3F_3)] &= 0.2075v_0 - 0.0520v_2 \\ V_T [NN(^3F_3), N\Delta(^5F_3)] &= 0.0696v_2 \\ V_D [N\Delta(^5P_3)] &= -0.1163v_0 + 0.0231v_2 \\ V_D [N\Delta(^3F_3)] &= 0.1418v_0 - 0.0067v_2 \quad \text{and} \\ V_D [N\Delta(^5F_3)] &= -0.1163v_0 - 0.0424v_2 . \end{aligned}$$

No phenomenological two-pion potentials are required. The non-zero fitted f -matrix parameters are $f_{NN,NN}^0 = 5.90$, $f_{N\Delta 2,N\Delta 2}^0 = 0.8$, $f_{N\Delta 1,N\Delta 1}^0 = 1.0$, and $f_{NN,N\Delta 2}^0 = -0.3$. As shown in Fig. 7, a very good fit is now obtained.

G. The $NN(^3F_4 - ^3H_4)$ System

In the case of the $NN(^3F_4 - ^3H_4)$ system the last published version dates back to the no-isobar results of Ref. [1]. In Ref. [3] it was remarked that coupling to isobars would not have a large effect for $T_L < 1 \text{ GeV}$ because of the large angular momentum barrier ($L \geq 3$). In fact, the change due to the couplings described below is not large, but there is a substantial improvement to the fit to $\delta(^3F_4)$.

The couplings to both the $N\Delta(^3F_4)$ and $N\Delta(^5F_4)$ channels are included, and the width of the Δ is not neglected in either channel. The potential matrix elements are

$$\begin{aligned} V_T [NN(^3F_4), N\Delta(^5F_4)] &= -0.116v_2 \\ V_T [NN(^3H_4), N\Delta(^5F_4)] &= 0.1032v_2 - 2.1 e^{-m_\pi r}/r \\ V_T [NN(^3F_4), N\Delta(^3F_4)] &= 0.207v_0 + 0.017v_2 \\ V_T [NN(^3H_4), N\Delta(^3F_4)] &= -0.078v_2 + 1.6 e^{-m_\pi r}/r \\ V_D [N\Delta(^5F_4)] &= -0.116v_0 - 0.0578v_2 \quad \text{and} \\ V_D [N\Delta(^3F_4)] &= 0.142v_0 + 0.0025v_2 . \end{aligned}$$

The strongly attractive OPE diagonal potential in the $N\Delta(^5F_4)$ state compensates substantially for the effect of the orbital angular momentum barrier, to the extent that too much low-energy coupling occurs. Consequently, a best fit requires the inclusion of a phenomenological two-pion range contribution in the $NN(^3H_4)$ coupling to $N\Delta(^5F_4)$ and $N\Delta(^3F_4)$, as above.

The fitted non-zero f -matrix constants are $f_{NNF,NNF}^0 = 8.49$, $f_{NNH,NNH}^0 = 30.0$, $f_{N\Delta 2,N\Delta 2}^0 = 5.0$, $f_{N\Delta 1,N\Delta 1}^0 = 5.0$, $f_{NNF,NNH}^0 = -2.0$, $f_{NNF,N\Delta 2}^0 = -3.90$, and $f_{NNF,N\Delta 1}^0 = 4.80$. The results, shown in Fig. 8, are in good agreement with the PSA except for $\delta(^3H_4)$. The predicted values of $\delta(^3H_4)$ are too attractive over most of the energy range. The coupling and the boundary condition have little effect because of the $L = 5$ barrier. If the PSA results are correct this appears to represent a deficiency of the potential in the NN sector for large L , perhaps indicating a repulsive quadratic spin-orbit term.

H. The $NN(^1G_4)$ System

As in the previous $NN(^3F_4 - ^3H_4)$ case, modified versions of the $NN(^1G_4)$ system have not been published since the uncoupled result of Ref. [1]. In this case the coupling is potentially more important because it couples to the $L = 2$ $N\Delta(^5D_4)$ channel. The width of that channel is included. The isobar related potential matrix elements are

$$\begin{aligned} V_T [NN(^1G_4), N\Delta(^5D_4)] &= 0.1663v_2 - 1.80 e^{-m_\pi r}/r \quad \text{and} \\ V_D [N\Delta(^5D_4)] &= -0.0383v_0 + 0.0553v_2 . \end{aligned}$$

A phenomenological two-pion range contribution in the coupling potential is again required to cancel a coupling effect which is too strong at medium-range. The f -matrix constants are $f_{NN,NN}^0 = 9.80$, $f_{N\Delta,N\Delta}^0 = 2.5$ and $f_{NN,N\Delta}^0 = 4.0$. As shown in Fig. 9 a very good fit to the PSA is obtained.

IV. Amplitudes and Observables

A. The Method

With respect to the observables, several conventions and notations are in use in the literature and it is sometimes difficult to determine their relationship. This is only partly due to notation in which zero-spin labels were omitted and triple-scattering experiments were not considered or clearly distinguished. It mainly owes its origin to the different definitions of the coordinate systems, both in the center of mass (c.m.) and in the laboratory. Here, we stick to the Madison convention (positive normal along $\vec{k}_i \times \vec{k}_f$) and the Saclay convention²¹ (same normal used in both frames but also in the three different sets of basis vectors in the laboratory) and avoid the Basle convention which uses two normals. Five common choices of orthonormal basis vectors are compared in Table II.

Table II

Group	Center of mass system						Laboratory system						
	k_i	k_f	n	ℓ	m	0	k	s	k'	s'	k''	s''	n
Saclay ²¹													
Wolfenstein ²²	p	p'	n	P	K	omitted	k		k'	s	k'_t	s_t	n
Halzen-Thomas ²³	z		y	\dagger	\ddagger	omitted	ℓ	s	ℓ	s	ℓ	s	n
Argonne ²⁴	same as above						L	S	L	S	L	S	N
Raynal ²⁵	k_i	k_f	n	P	K	omitted	z	$-x$					y

A selection of some representative sets of basis vectors encountered in the literature both in the center-of-mass and laboratory systems. The vectors listed in a column are all equal. One notes that (\dagger) $\ell = x \sin \frac{\theta}{2} + z \cos \frac{\theta}{2}$ and (\ddagger) $m = x \cos \frac{\theta}{2} - z \sin \frac{\theta}{2}$.

The notation (Beam, Target; Scattered, Recoil) to express observables is adopted by Argonne whereas Saclay used (Scattered, Recoil; Beam, Target). The position of a given spin label in such a quadruplet thus specifies to which particle the frame is attached. Taking into account this information, the use of superscripts in the lab system by Saclay, in order to distinguish the incident

(no superscript), scattered (prime) and recoil (double dash) particle is unnecessary and is being progressively abandoned in observable names. It follows from Table II that $C_{KK} \equiv C_{mm00} = A_{00mm}$, $C_{PP} \equiv C_{\ell\ell00} = A_{00\ell\ell}$ and $C_{KP} = C_{PK} \equiv C_{\ell m00} = -A_{00\ell m}$ (in each case the last equality comes from time reversal invariance). The Wolfenstein system²² is then simply connected to the Saclay system.²¹

The theoretical group at Argonne is used to the Halzen-Thomas notation.²³ Their vectors (x, y, z) define the center-of-mass helicity frame. The experimental group at Argonne has capitalized the spin labels in the laboratory system. Raynal's notation²⁵ defines the spin labels x, y, z in the laboratory system but these should not be confused with the Argonne center-of-mass same labels x, y, z . The polarization labels X, Y, Z used in the SAID system correspond to Raynal's index x, y, z . Convention also differs, as the sideways direction \vec{x} is chosen to be parallel to $-\vec{s}$. Such a choice, however, is not consistent with the Saclay right-handed frame attached to the incident particle in the lab system and for which $\vec{n} = \vec{k} \times \vec{s}$. This is the reason why some minus signs appear when connecting Raynal's formalism²⁵ to the Saclay one, *e.g.* $A_{ZX} = -A_{00sk}$. Still another different convention but now in the center of mass can be found in the literature. In some older papers, the defined vector \vec{m} has its sign opposite to the one of the Saclay formalism. This was made so that in the non-relativistic case the c.m. vector \vec{m} coincides in direction with the lab momentum \vec{k}'' of the recoil particle, which is not the case in the Saclay formalism. Again connecting identical observables defined with respect to two different triplets of basis vectors in the same coordinate system involves a few minus signs (as many as there are in index). A final source of confusion is that the Wolfenstein transfer parameters R'_t and A'_t are defined with respect to $-\vec{k}''$, which changes the sign of the normal in the lab frame attached to the recoil particle. The connection with the Saclay formalism is then $R'_t = -K_{0k''s0}$ and $A'_t = -K_{ok''k0}$.

Keeping these five choices in mind, we compare the observables that we will compute, in Table III. One can see that the names used in the SAID system come from different groups.

We calculate the observables from the bilinear combinations of the helicity amplitudes, which themselves are constructed from the partial-wave S -matrix. The spin partial-wave expansions of

Table III

SAID ¹⁸	Wolfenstein ²²	Saclay ²¹	Argonne ²⁴
<i>DSG</i>	I_0	σ or $\frac{d\sigma}{d\Omega}$	σ
<i>P</i>	P	$P_{n000} = A_{00n0} = A_{000n}$	P
<i>D</i>	D	$D_{n0n0} = D_{0n0n}$	D_{NN}
<i>DT</i>	D_t	$K_{n00n} = K_{0nn0}$	K_{NN}
<i>R</i>	R	$D_{s'0s0} = D_{0s''0s}$	D_{SS}
<i>RP</i>	R'	$D_{k'0s0} = D_{0k''0s}$	D_{SL}
<i>A</i>	A	$D_{s'0k0} = D_{0s''0k}$	D_{LS}
<i>AP</i>	A'	$D_{k'0k0} = D_{0k''0k}$	D_{LL}
<i>AYY</i>	C_{nn}	$C_{nn00} = A_{00nn}$	C_{NN}
<i>AZZ</i>	$C_{PP} \cos^2 \frac{\theta}{2} + C_{KK} \sin^2 \frac{\theta}{2} + C_{KP} \sin \theta$	A_{00kk}	C_{LL}
<i>AXX</i>	$C_{PP} \sin^2 \frac{\theta}{2} + C_{KK} \cos^2 \frac{\theta}{2} - C_{KP} \sin \theta$	A_{00ss}	C_{SS}
<i>AZX</i>	$\frac{1}{2}(C_{KK} - C_{PP}) \sin \theta + C_{KP} \cos \theta$	$-A_{00sk} = -A_{00ks}$	$-C_{SL}$

Name and sign conventions for the observables computed in the present paper in relation to four different formalisms often encountered in the literature.

the s -channel helicity amplitudes follow:

$$\phi_1 = \frac{1}{2ik} \sum_{J=0}^{\infty} \left\{ (2J+1)\alpha_{J,0} + (J+1)\alpha_{J,+} + J\alpha_{J,-} + 2\sqrt{J(J+1)}\alpha^J \right\} P_J(\cos \theta) \quad (4.1a)$$

$$\phi_2 = \frac{1}{2ik} \sum_{J=0}^{\infty} \left\{ -(2J+1)\alpha_{J,0} + (J+1)\alpha_{J,+} + J\alpha_{J,-} + 2\sqrt{J(J+1)}\alpha^J \right\} P_J(\cos \theta) \quad (4.1b)$$

$$\begin{aligned} \phi_3 = \frac{1}{2ik} \sum_{J=0}^{\infty} & \left\{ (2J+1)\alpha_{J,1} + J\alpha_{J,+} + (J+1)\alpha_{J,-} - 2\sqrt{J(J+1)}\alpha^J \right\} \times \\ & \times \left[P_J(\cos \theta) + \frac{1 - \cos \theta}{J(J+1)} P'_J(\sin \theta) \right] \end{aligned} \quad (4.1c)$$

$$\begin{aligned} \phi_4 = \frac{1}{2ik} \sum_{J=0}^{\infty} & \left\{ -(2J+1)\alpha_{J,1} + J\alpha_{J,+} + (J+1)\alpha_{J,-} - 2\sqrt{J(J+1)}\alpha^J \right\} \times \\ & \times \left[-P_J(\cos \theta) + \frac{1 + \cos \theta}{J(J+1)} P'_J(\sin \theta) \right] \end{aligned} \quad (4.1d)$$

$$\phi_5 = \frac{1}{2ik} \sum_{J=0}^{\infty} \left\{ \sqrt{J(J+1)} (\alpha_{J,+} - \alpha_{J,-}) - \alpha^J \right\} \frac{\sin \theta P'_J(\sin \theta)}{\sqrt{J(J+1)}} \quad (4.1e)$$

where $P_J(\cos \theta)$ are the Legendre polynomials of order J (total angular momentum), $P'_J(\sin \theta)$ are derivatives of $P_J(\cos \theta)$ with respect to their argument, and k is the c.m. relative momentum. The energy-dependent α parameters describe the different possible S - and L -coupled states: $\alpha_{J,0}$ represents the S singlet, $\alpha_{J,1}$ represents the uncoupled triplet, and $\alpha_{J,\pm}$, α^J represent the L coupled triplet states. It is understood in the above sums that, when we encounter a $J = 0$ pole, then the corresponding α parameter has to be set equal to zero. Consequently, $\alpha_{0,1} = \alpha_{0,+} = \alpha_{0,-} = \alpha^0 = 0$ and the 3P_0 lacks a coupled partner. The $\alpha_{J,\pm}$ are the diagonal partial-wave amplitudes for tensor coupled states while the α^J are the off-diagonal ones.

If δ is the real part of the complex phase shift and $\sqrt{\eta}$ is the exponential of the imaginary part, then our inelasticity parameter η is related to Bugg's ρ [Ref. 17] by the equality $\eta = \cos^2 \rho$. Thus $\eta = 1$ in the elastic region $T_L \lesssim 300$ MeV. For the parametrization of the partial-wave amplitudes in terms of the phase shift parameters, that definition leads to

$$\alpha_{J,0} = \eta_{J,0} \exp(2i\delta_{J,0}) - 1 \quad (4.2a)$$

$$\alpha_{J,1} = \eta_{J,1} \exp(2i\delta_{J,1}) - 1 \quad (4.2b)$$

$$\alpha_{J,\pm} = \eta_{J,\pm} \cos 2\bar{\epsilon}_J \exp(2i\delta_{J,\pm}) - 1 \quad (4.2c)$$

$$\alpha^J = i\sqrt{\eta_{J,-}\eta_{J,+}} \sin 2\bar{\epsilon}_J \exp[i(\delta_{J,-} + \delta_{J,+} + \phi_J)] \quad (4.2d)$$

where $\eta_{J,\pm} = 1$ in the elastic region and $\bar{\epsilon}_J$ is the elastic (or "real") mixing angle between $J,+/-$ states. The inelastic (or "imaginary") mixing angle ϕ_J is non zero only above the inelastic threshold and its numerical value remains small just above it. However, for states of small angular momentum barrier (*e.g.* ϕ_2), its value rises rapidly with energy and has a point of inflection at the Δ resonance threshold. The mixing angles ϕ_2 and ϕ_4 presented in relation (4.2d) should not be confused with the helicity amplitudes of eqs. (4.1b) and (4.1d). In that representation, the $\bar{\epsilon}$'s, δ 's and ϕ 's are real numbers and $0 \leq \eta \leq 1$. In our figures for η 's and δ 's, we replace the set of label subscripts by the spectroscopic notation $({}^{2S+1}L_J)$. The coupled channel parameters obey a sub-unitarity inequality²⁶ because of their other coupling to isobar channels. Finally, the 3P_0 wave has the same representation as the singlet states.

The observables can be computed from the c.m. invariant amplitudes used by Saclay (formulae

can be found in Ref. [21]) and related to the helicity amplitudes by

$$a = \frac{1}{2} [(\phi_1 + \phi_2 + \phi_3 - \phi_4) \cos \theta - 4\phi_5 \sin \theta] \quad (4.3a)$$

$$b = \frac{1}{2} (\phi_1 - \phi_2 + \phi_3 + \phi_4) \quad (4.3b)$$

$$c = \frac{1}{2} (-\phi_1 + \phi_2 + \phi_3 + \phi_4) \quad (4.3c)$$

$$d = \frac{1}{2} (\phi_1 + \phi_2 - \phi_3 + \phi_4) \quad (4.3d)$$

$$e = -\frac{i}{2} [(\phi_1 + \phi_2 + \phi_3 - \phi_4) \sin \theta + 4\phi_5 \cos \theta] \quad (4.3e)$$

This particular set of amplitudes is transformed into several other ones also used in the literature, in Refs. [21] and [27]. The normalization is such that

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \{ |a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2 \} \quad (4.4)$$

is in mb/sr . For $J \leq 4$, the partial waves used are the coupled channel ones of Section III, that is 1S_0 , 3P_0 , 3P_1 , $^3P_2 - ^3F_2$, 1D_2 , 3F_3 , $^3F_4 - ^3H_4$ and 1G_4 . In fitting the energy dependence of the total cross sections, a compromise is reached in using the waves 3H_5 and 1I_6 given by the Feshbach-Lomon model without isobar coupling.⁴ The same NN coupling constant in the π - and 2π - sector (14.40) is used throughout. The unitarized OPE phases begin with $^3H_6 - ^3J_6$, $^3J_7, \dots$ until $J = 12$, the starting value for the OPE Born amplitude. The Coulomb interaction is included, except for the total cross sections obtained by the optical theorem. In the latter case, the pure Coulomb amplitudes and phases are removed but the electro-nuclear interferences in the model phases are kept.

The above prescriptions for treating OPE and Coulomb effects in our model are of course slightly different than those of Bugg¹⁷ or Arndt.¹⁸ Nevertheless, we will see that the different treatments lead to minor differences in the polarization observables at low energy (142 MeV) and that the differences become appreciable only when $\theta \lesssim 5^\circ$ and only at higher energies. The observables of Table III are calculated at energies of 142, 210, 325, 400, 475, 515, 580, 615, 685, 750, 800 and 970 MeV. The excitation functions to be displayed below cover the whole range of energies, $T_L \leq 1$ GeV, whereas angular distributions are presented at 142, 515 and 800 MeV only (except for A_{zz} for which we add 580 and 685 MeV).

B. The Results

The excitation curves for the total unpolarized (σ_{TOT}) and polarized ($\Delta\sigma_L$ and $\Delta\sigma_T$) cross sections are shown in Figure 10. The experimental points are in general well reproduced by the model predictions as the latter follow the correct curvatures and structures in the energy dependence, especially for $\Delta\sigma_L$. However $\Delta\sigma_T$ does deviate from the data in that the position of the model peak is about 50 MeV (laboratory energy) higher than the data peak, and the prediction falls off more slowly than the data after the peak. This is correlated with the deviation, already noted in Section III, of $\eta(^1D_2)$ from the PSA result for $T_L > 600$ MeV. The model of $\eta(^1D_2)$ continues to decrease after the PSA result levels off at 600 MeV, and it does not rise to the PSA value until $T_L \geq 800$ MeV. Because $\Delta\sigma_T$ increases with decreasing $\eta(^1D_2)$, the model peak is shifted to higher energy and the value is higher than the experimental value until $T_L \approx 900$ MeV.

The excitation functions at 90° c.m. for $\frac{d\sigma}{d\Omega}$, R , R' , A , A' , D , C_{nn00} , A_{00kk} , A_{00ss} and C_{KP} are compared with the data points between 89° and 91° compiled in the SAID system,¹⁸ and are displayed in Figure 11. Our predictions for C_{KK} are also compared with Arndt PSA results. For identical particles at $\theta_{cm} = \frac{\pi}{2}$, other theoretical predictions are $P = 0$, $A_{00sk} = 0$ and $D_{n0n0} = K_{0nn0}$ (the few existing D_t experimental points are thus plotted together within the D experimental set). We note that R' and A' are not independent quantities at this angle. For pp scattering at 90° , one has $\frac{A}{A'} = -\frac{R'}{R} = \tan\theta_1$ with θ_1 being the lab scattering angle. Theory implies that the polarization-transfer parameters here are not independent from the depolarization and rotation parameters since $R_t(90^\circ) = R(90^\circ)$, $R'_t(90^\circ) = R'(90^\circ)$, $A_t(90^\circ) = -A(90^\circ)$ and $A'_t(90^\circ) = -A'(90^\circ)$. Consequently we could have plotted any independent measurements of R_t , R'_t , A_t , A'_t together with the usual Wolfenstein set, as we did for D_t and D . However, at this time no data points exist for the transfer parameters in the range $88^\circ < \theta < 92^\circ$ up to 1 GeV. Under the same circumstances, one also has $A_{00kk} + A_{00ss} = C_{nn00} - 1$ because of the Pauli principle. Other constraints at 90° c.m. on the limits of some spin observables from amplitude analysis can be found in Ref. [28].

The fit of the 90° excitation functions to the data is generally very good. The only substantial deviation from data for $T_L \leq 800$ MeV is for $\frac{d\sigma}{d\Omega}(90^\circ)$ at $650 \text{ MeV} < T_L \leq 800 \text{ MeV}$. In this range the newest data is higher than the older data and our prediction. Above 800 MeV, up to 1 GeV, our predictions for D , A_{yy} and A_{zz} decrease with energy while the data remains nearly constant.

For the R' , A' , A_{xx} and C_{KK} excitation functions we also show the result of the SAID PSA¹⁸ because the data is sparse or non-existent. The trend is similar to our prediction, with somewhat less structure.

The comparison with the angular distributions is shown in Fig. 12. The model energies chosen as representative are $T_L = 142, 515$ and 800 MeV for $\frac{d\sigma}{d\Omega}$, P , R , R' , A , A' , D , D_t , A_{yy} , A_{zz} , A_{zx} , and A_{xx} . In addition we compare with recent A_{zz} data (LAMPF) at 580 and 685 MeV. Again the Arndt PSA prediction¹⁸ is also shown where the data is sparse or non-existent. The experimental energies compared with these curves are listed in the caption. They are mostly within 5 MeV of the model energy, but sometimes differ by as much as 10 MeV. There also exists some Wolfenstein spin-transfer R_t and A_t data,¹⁸ but only at 800 MeV. There are eight points of moderate to poor accuracy. None of the points are near $\theta_{cm} = 90^\circ$. We have not calculated the model predictions for R_t and A_t or any of the triple-index spin observables.

The fit to the angular distributions is good, and has no important deviations from the data for $T_L < 800$ MeV. Most of the observables are also well reproduced at $T_L = 800$ MeV. However at this energy the model shows more angular dependence than the data for the observables P , R' , A , A_{yy} , and A_{zz} . This trend persists at 580 and 685 MeV for A_{zz} , but the statistical significance is small. The problem may be associated with the model value of $\delta(^3H_4)$. This has the only significant deviation from the PSA of any of the δ 's. The high L value implies a strong angular dependence, and our $\delta(^3H_4)$ is significantly larger than the PSA for $T_L > 500$ MeV.

V. Conclusions

We have presented the results of a model for pp scattering that includes:

- (a) Coupled NN , $N\Delta$ and $\Delta\Delta$ channels for $J \leq 4$. The effect of the width of the Δ is taken into account.
- (b) The π , 2π , η , ρ and ω meson exchange potentials in the NN sector, and π -exchange potentials connecting the isobar channel sector to itself and to the NN sector. The coupling parameters used were determined by experiments other than NN scattering or from $SU(3)$ and quark model relations.

- (c) At a separation radius r_0 an R -matrix boundary condition is imposed representing valence quark degrees of freedom for $r < r_0$. The energy independent part of the f -matrix are parameters. The poles are determined by the energies of the quark configurations as given by the CBM. Only the poles of the lowest 6-quark configuration $(1s_{\frac{1}{2}})^6$ are included, affecting the 1S_0 and 1D_2 pp channels. The value of r_0 is determined by the physical requirements of the R -matrix method.

The result is a very good fit to almost all the data for $T_L < 1$ GeV. The major exceptions are the higher energy of the peak in $\Delta\sigma_T$ and an excessive angular dependence in a few of the observables. No other model produces quality fits for $T_L > 300$ MeV and they especially fail in describing the structures in $\Delta\sigma_L$ and $\Delta\sigma_T$. The good degree to which these structures are fitted by this model strongly implies that the structures are due to the effect of coupling to the $N\Delta$ threshold in the NN 1D_2 and 3F_3 channels.

The shift of the $\Delta\sigma_T$ peak is related to the fact that the model $\eta(^1D_2)$ structure is deeper than the PSA result. The inability of the model to more closely fit the PSA value of $\eta(^1D_2)$ may be related to the omission in the model of coupling to the πD channel. This channel couples relatively strongly to the $NN(^1D_2)$. Including it, in addition to the isobar channel coupling of the model, may allow a simultaneous fit of the steep descent of $\eta(^1D_2)$ just above threshold and the weakened inelastic effect above 650 MeV laboratory energy.

The excess of angular structure in some of the observables for $T_L > 515$ MeV is likely due to the model value of $\delta(^3H_4)$ being larger than the PSA value at the higher energies. The difference is at most 0.5° , but this can be significant because of the high statistical weight of the $J = 4$ partial wave. Because of the very large angular momentum barrier ($L = 5$) the boundary condition parameter has negligible effect. This indicates that the medium range meson-exchange potential used here is inaccurate for large L . A correction to our potential can arise from high order non-local effects which give rise to quadratic spin-orbit type terms. These were not included.

In spite of the above noted deficiencies the pp data for $T_L < 1$ GeV is overall well understood in terms of this model implying that most of the physics has been included. An important part of this physics is the R -matrix boundary condition representing the quark degrees of freedom at short range. Even in those channels in which we ignore the pole terms (because they are distant), the

energy independent boundary condition acts differently than any energy-independent non-singular potential.⁴ In fact it behaves as if the interior potential increases linearly with energy, asymptotically becoming a hard core²⁹, and the present results reinforce that conclusion.

Of course the most interesting effects of the R -matrix core representation come from the poles coinciding with quark configuration energies. These give rise to exotic quark resonances, exotic dibaryons in the present case, which begin at $T_L > 1.8$ GeV. The implications for structure in the observables have already been given for the earlier forms of these model^{2,3} and have interesting correlations with experiment.^{3,30,31} Those predictions will soon be updated with the present models, and the models themselves will be extended to include the poles due to the $(1s_{\frac{1}{2}})^5(1p_{\frac{1}{2}})$ configuration. This modification to the odd-parity states will negligibly affect the $T_L < 1$ GeV predictions.

The other obvious extension of this work is to update the $I = 0$ partial wave models and compare with the np data. That work is well underway.

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FIGURE CAPTIONS

Fig. 1: Meson-exchange potential diagrams. (a)–(c) Contributions to NN - NN sector potentials; (d) for NN - N -isobar potentials; (e) for NN - $\Delta\Delta$ potentials; (f) for $N\Delta$ - $N\Delta$ potentials; (g) for $N\Delta$ - ΔN potentials.

Fig. 2: Phase parameters for NN (1S_0) scattering. Model – solid curve; PSA of Ref. [18] – dashed curve; PSA of Ref. [17] – \square .

Fig. 3: Phase parameters for NN (3P_0) scattering. Notation as in Fig. 2.

Fig. 4: Phase parameters for NN (3P_1) scattering. Notation as in Fig. 2.

Fig. 5: Phase parameters for the tensor coupled NN ($^3P_2 - ^3F_2$) scattering. The parameters determine the S -matrix

$$\begin{aligned} S_{--} &= \eta \left({}^3(J-1)_J \right) \cos 2\bar{\epsilon}_J \exp 2i\delta \left({}^3(J-1)_J \right) , \\ S_{+-} = S_{-+} &= i \left[\eta \left({}^3(J-1)_J \right) \eta \left({}^3(J+1)_J \right) \right]^{1/2} \sin 2\bar{\epsilon}_J \\ &\quad \times \exp \left[\delta \left({}^3(J-1)_J \right) + \delta \left({}^3(J+1)_J \right) + \phi_J \right] , \\ S_{++} &= \eta \left({}^3(J+1)_J \right) \cos 2\bar{\epsilon}_J \exp 2i\delta \left({}^3(J+1)_J \right) . \end{aligned}$$

Notation as in Fig. 2.

Fig. 6: Phase parameters for NN (1D_2) scattering. Notation as in Fig. 2.

Fig. 7: Phase parameters for NN (3F_3) scattering. Notation as in Fig. 2.

Fig. 8: Phase parameters for the tensor coupled NN ($^3F_4 - ^3H_4$) scattering. Parameters determine S -matrix as in Fig. 5. Notations as in Fig. 2.

Fig. 9: Phase parameters for NN (1G_4) scattering. Notation as in Fig. 2.

Fig. 10: Excitation functions for the total unpolarized (σ_{TOT}) and polarized ($\Delta\sigma_L$ and $\Delta\sigma_T$) pp cross sections for $T_L < 1$ GeV.

\square : experimental values compiled in SAID.¹⁸

\diamond for $\Delta\sigma_T$: Saclay points (PE86).

Solid curves: model.

Fig. 11: Excitation functions at $\theta_{cm} = 90^\circ$ for the differential cross section $\frac{d\sigma}{d\Omega}$, Wolfenstein parameters R, R', A, A', D , second-rank asymmetry spin-tensor A_{yy}, A_{zz}, A_{xx} ($A_{zx} = 0$ at this angle), and spin-correlations C_{KP} and C_{KK} . The two last parameters are defined in the c.m. frame but the experimental points shown do represent independent measurements. Note that $C_{PP}(90^\circ) = C_{KK}(90^\circ)$.

\square : experimental values compiled in SAID.

\times for $\frac{d\sigma}{d\Omega}$: Saclay points (GA85).

\times for D : independent D_t points used since $D(90^\circ) = D_t(90^\circ)$ (they are listed in SAID¹⁸).

Solid curves: model.

Dashed curves: Arndt's PSA predictions.¹⁸

Fig. 12: Angular distributions computed at $T_L = 142, 515$, and 800 MeV compared with experimental values compiled in SAID¹⁸ around each energy, for the following observables.

$\frac{d\sigma}{d\Omega}$ at 137 (\diamond), 144 (\circ) and 144.1 (\times) MeV

513 (\square) and 516 (\times) MeV

788.7 (\diamond), 789 (\times), 795 (\square) and 800 (\circ) MeV.

P at 137 (\diamond), 140.7 (\square), 142 (\circ) and 147 (\times) MeV

515 (\square), 515.3 (\diamond) and 517 (\times) MeV

790.1 (\diamond), 794 (\times), 796 (\square) and 800.9 (\circ) MeV.

R at 140 (\square) and 142 (\circ) MeV

515.3 (\square) and 517 (\times) MeV

800 (\circ, \square, \diamond) MeV.

R' at 137.5 (\square) and 140.4 (\circ) MeV

515.3 (\square) and 520 (\times) MeV

800 ($\times, \square, \diamond$) MeV.

A at 139 (\square) and 143 (\circ) MeV

517 (\square) MeV

800 (\circ, \square, \diamond) MeV.

A' at 800 ($\times, \square, \diamond$) MeV.

D at 138 (\square), 142 (\circ) and 143 (\times) MeV

515.3 (\square) and 517 (\times) MeV

800 ($\circ, \times, \square, \diamond$) MeV.

D_t at 517 (\square) MeV

800 (\circ, \square) MeV.

A_{xx} at 514 (\square) MeV

791 (\circ) MeV.

$-A_{zx}$ at 514 (\square) MeV

793 (\times), 794 (\circ) and 805.7 (\square) MeV.

A_{zz} at 514 (\square) and 518.4 (\times) MeV

577 (\times), 583 (\circ), 586.3 (\diamond) and 589 (\square) MeV

688 (\times) and 692 (\square) MeV

790.1 (\diamond), 793 (\times), 800 (\circ) and 805.7 (\square) MeV.

A_{yy} at 143 (\square) MeV

515 (\square) MeV

790.1 (\diamond), 794 (\times), 796 (\square) and 800 (\circ) MeV.

Curves notated as in Fig. 11.

For a copy of figure 1
[the only one which is not in PostScript]
please send an email request
containing your mailing address
to ereidell@marie.mit.edu

Thank you!
--- Evan A. Reidell (MIT CTP support staff)







































